Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WME01/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Mechanics M1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$, and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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1.

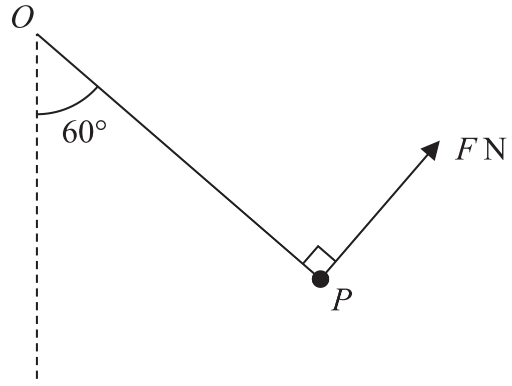


Figure 1

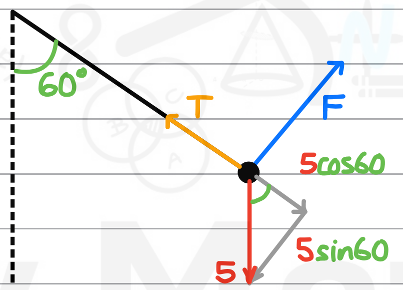
A particle P of weight 5 N is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point O. The particle P is held in equilibrium by a force of magnitude F newtons. The direction of this force is perpendicular to the string and OP makes an angle of 60° with the vertical, as shown in Figure 1.

Find

(a) the value of F **(3)**

(b) the tension in the string. **(3)**

a) Redraw the diagram labelling forces.



The particle is in equilibrium so the sum of the forces parallel and perpendicular to the string is zero.

Perpendicular : $F - 5\sin 60 = 0$ $\quad F = 5\sin 60 = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$ N

b) Parallel : $T - 5\cos 60 = 0$ $\quad T = 5\cos 60 = 5 \times \frac{1}{2} = \frac{5}{2}$ N

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Question 1 continued

Q1

(Total 6 marks)

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2. A particle P has mass km and a particle Q has mass m. The particles are moving towards each other in opposite directions along the same straight line when they collide directly.

Immediately before the collision, P has speed $3u$ and Q has speed u.

As a result of the collision, the direction of motion of each particle is reversed and the speed of each particle is halved.

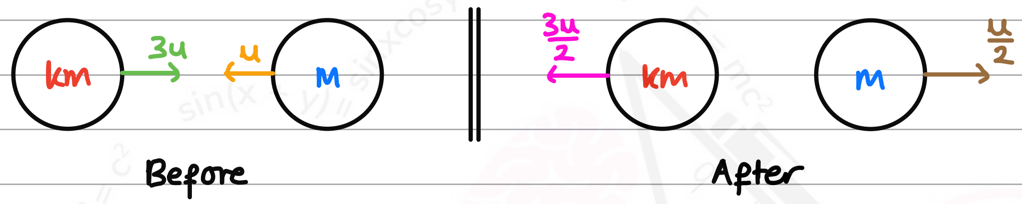
(a) Find the value of k.

(4)

(b) Find, in terms of m and u, the magnitude of the impulse exerted on Q in the collision.

(3)

a) Draw a diagram showing masses and speeds before and after.



Using the conservation of momentum formula : $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore km(3u) + m(-u) = km\left(-\frac{3u}{2}\right) + m\left(\frac{u}{2}\right)$

$3ukm - mu = -\frac{3}{2}ukm + \frac{1}{2}mu$

$3k - 1 = -\frac{3}{2}k + \frac{1}{2}$

$3k + \frac{3}{2}k = \frac{1}{2} + 1 \quad \therefore \frac{9}{2}k = \frac{3}{2} \quad \therefore k = \frac{1}{3}$

b) Impulse formula : $I = mv - mu$

$\therefore I = m\left(\frac{u}{2}\right) - m(-u) = \frac{3}{2}mu \qquad \therefore |I| = \frac{3}{2}mu$

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Question 2 continued

Q2

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3.

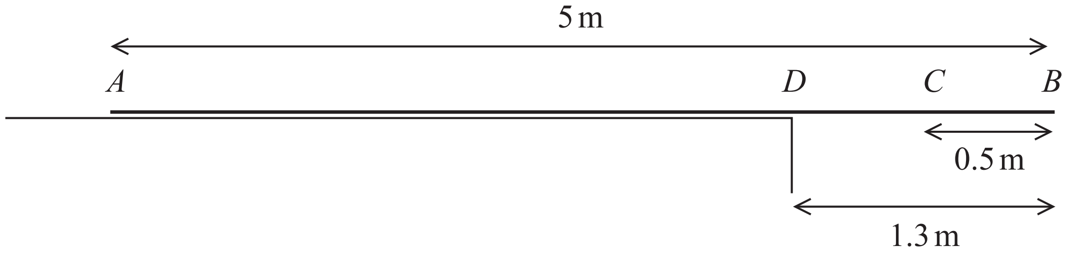


Figure 2

A beam *ADCB* has length 5 m. The beam lies on a horizontal step with the end *A* on the step and the end *B* projecting over the edge of the step. The edge of the step is at the point *D* where *DB* = 1.3 m, as shown in Figure 2.

When a small boy of mass 30 kg stands on the beam at *C*, where *CB* = 0.5 m, the beam is on the point of tilting.

The boy is modelled as a particle and the beam is modelled as a uniform rod.

(a) Find the mass of the beam.

(3)

A block of mass *X* kg is now placed on the beam at *A*.

The block is modelled as a particle.

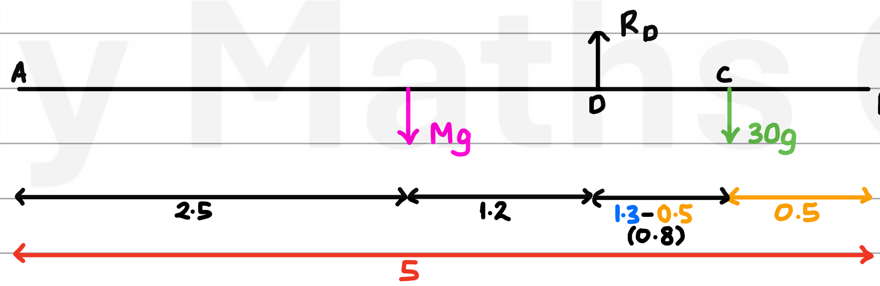
(b) Find the smallest value of *X* that will enable the boy to stand on the beam at *B* without the beam tilting.

(3)

(c) State how you have used the modelling assumption that the block is a particle in your calculations.

(1)

a) Draw the diagram labelling the relevant forces.



Since it's stated in the question that the beam is in equilibrium, the sum of the clockwise moments is equal to the sum of the anticlockwise moments, therefore :

Σ moments clockwise = Σ moments anticlockwise where moment = force × perpendicular distance

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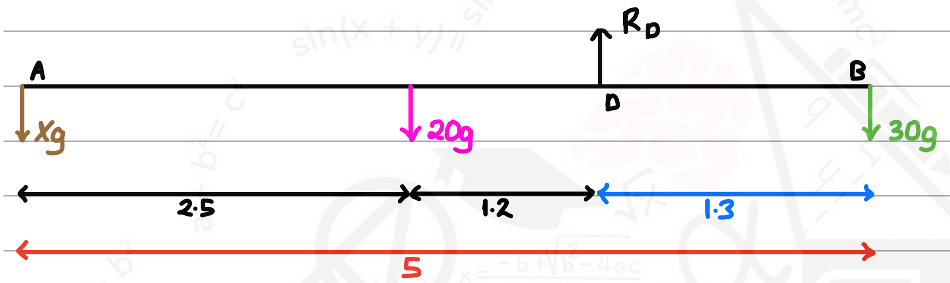
Question 3 continued

The clockwise forces are ones that go upwards from the left or downwards from the right of where moments are taken and anticlockwise forces are ones that go upwards from the right or downwards from the left of where moments are taken.

Taking moments around D: $\underbrace{Mg \times 1.2}_{\text{anticlockwise moments}} = \underbrace{30g \times 0.8}_{\text{clockwise moments}}$

$\therefore M = \frac{0.8 \times 30g}{1.2g} = 20 \text{ kg}$

b) Redraw the diagram labelling the new forces.



If the beam isn't going to tilt with the block placed at A, the beam is on the point of tilting at D and the boy must stand at B to counteract the moment created by the block by the greatest amount.

Taking moments around D: $\underbrace{Xg \times 3.7 + 20g \times 1.2}_{\text{anticlockwise moments}} = \underbrace{30g \times 1.3}_{\text{clockwise moments}}$

$\therefore 3.7Xg + 24g = 39g \qquad \therefore X = \frac{39g - 24g}{3.7g} = \frac{150}{37} \approx 4.05 \text{ kg}$ (3sf)

c) The weight of the block acts at a single point.

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Question 3 continued

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Question 3 continued

Q3

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4. At time $t = 0$, a small ball is projected vertically upwards from a point A which is 24.5 m above the ground. The ball first comes to instantaneous rest at the point B, where $AB = 19.6$ m and first hits the ground at time $t = T$ seconds.

The ball is modelled as a particle moving freely under gravity.

(a) Find the value of T.

(6)

(b) Sketch a speed-time graph for the motion of the ball from $t = 0$ to $t = T$ seconds.

(No further calculations are needed in order to draw this sketch.)

(2)

a) The question states that the ball is modelled as a particle that falls freely under gravity meaning the acceleration is constant so SUVAT can be used. Since there are two unknowns, we need to form two equations and solve them simultaneously to calculate T.

SUVAT from A to B (Positive as upwards):

s : 19.6
u : u
v : 0
a : -9.8
t :

Equation without time: $v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times -9.8 \times 19.6$

$u^2 = 384.16 \qquad u = 19.6\,\text{ms}^{-1}$

SUVAT from A to the ground (Positive as upwards):

s : -24.5
u : 19.6
v :
a : -9.8
t : T

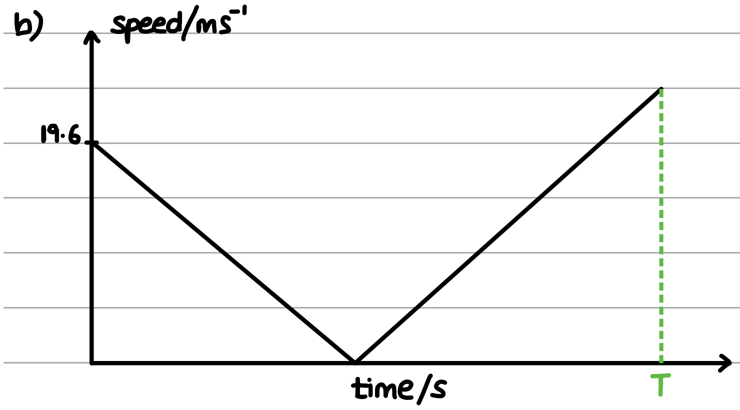
Equation without v: $s = ut + \frac{1}{2}at^2$

$-24.5 = 19.6(T) + \frac{1}{2}(-9.8)(T^2)$

$-4.9T^2 + 19.6T + 24.5 = 0$

$T = 5$ ✓, $T = -1$ ✗ $(T > 0)$ $\therefore T = 5$

b)



The initial speed was 'u' which was calculated to be 19.6 ms⁻¹ however isn't required to be labelled. The final speed must be more than the initial speed to conserve energy.

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total 8 marks)

Q4

5.

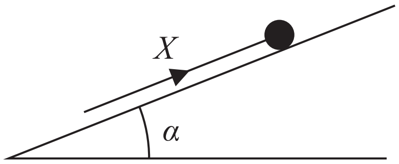


Figure 3

A particle of mass m rests in equilibrium on a fixed rough plane under the action of a force of magnitude X. The force acts up a line of greatest slope of the plane, as shown in Figure 3.

The plane is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

The coefficient of friction between the particle and the plane is μ.

•① When $X = 2P$, the particle is on the point of sliding up the plane.

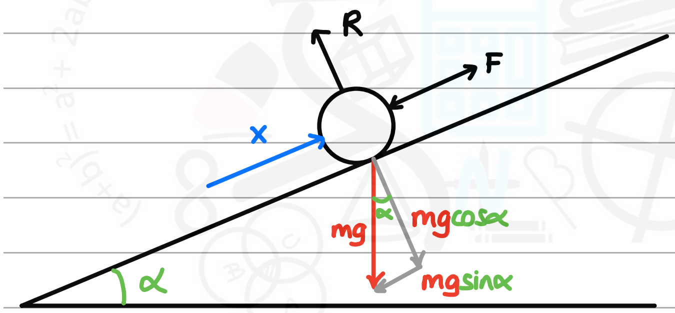
•② When $X = P$, the particle is on the point of sliding down the plane.

Find the value of μ.

(8)

Redraw the diagram labelling the forces.

$\tan\alpha = \frac{3}{4}$, $\sin\alpha = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$



Since there's no movement perpendicular to the plane, the sum of the forces perpendicular to the plane is zero.

$\Sigma F = 0$ $\therefore R - mg\cos\alpha = 0$

$\therefore R = mg\cos\alpha$

In situation ①, the particle is on the point of sliding up the plane so friction acts in the opposite direction down the plane and remains at rest so the sum of the parallel forces is zero. Taking up the slope as positive :

$\Sigma F = 0$ $\therefore X - F - mg\sin\alpha = 0$ $\therefore X = F + mg\sin\alpha$ where $X = 2P$

For situation ②, the opposite effect is taking place so friction acts up the plane because the particle wants to slide down the slope. The particle remains at rest so the sum of the parallel forces is zero. Taking up the slope as positive :

$\Sigma F = 0$ $\therefore X + F - mg\sin\alpha = 0$ $\therefore X = mg\sin\alpha - F$ where $X = P$

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Question 5 continued

$F = \mu R$ so we can rewrite the equations as :

$2P = \mu R + mg\sin\alpha$ $\quad P = mg\sin\alpha - \mu R$, replacing X with their corresponding value.

$\therefore P = \frac{1}{2}(\mu R + mg\sin\alpha)$ $\quad \therefore mg\sin\alpha - \mu R = \frac{1}{2}(\mu R + mg\sin\alpha)$

$\therefore 2mg\sin\alpha - 2\mu R = \mu R + mg\sin\alpha$ Substitute $mg\cos\alpha$ for R.

$\therefore 2mg\sin\alpha - 2\mu mg\cos\alpha = \mu mg\cos\alpha + mg\sin\alpha$

$\therefore 2\sin\alpha - 2\mu\cos\alpha = \mu\cos\alpha + \sin\alpha$

$\therefore 3\mu\cos\alpha = \sin\alpha$

$\mu = \frac{\sin\alpha}{3\cos\alpha} = \frac{1}{3}\tan\alpha = \frac{1}{3}\left(\frac{3}{4}\right) = \frac{1}{4} = 0.25$ //

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Question 5 continued

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Question 5 continued

(Total 8 marks)

Q5

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6. [*In this question* **i** *and* **j** *are horizontal unit vectors*.]

A particle P of mass 2 kg moves under the action of two forces, $(p\mathbf{i} + q\mathbf{j})$ N and $(2q\mathbf{i} + p\mathbf{j})$ N, where p and q are constants.

Given that the acceleration of P is $(\mathbf{i} - \mathbf{j})$ m s^{-2}

(a) find the value of p and the value of q.

(5)

(b) Find the size of the angle between the direction of the acceleration and the vector **j**.

(2)

At time $t = 0$, the velocity of P is $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}

At $t = T$ seconds, P is moving in the direction of the vector $(11\mathbf{i} - 13\mathbf{j})$.

(c) Find the value of T.

(5)

a) We are given mass, force and acceleration and the equation linking these is :

$\Sigma F = ma$

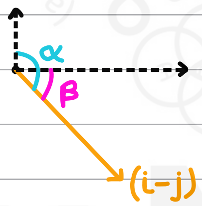
$\therefore (pi + qj) + (2qi + pj) = 2(i - j)$

$\therefore (2q + p)i + (q + p)j = 2i - 2j$

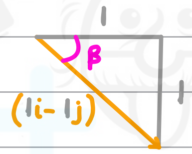
– Equate i and j components

$\therefore 2q + p = 2, \quad q + p = -2 \quad \therefore p = -6, \; q = 4$

b) Draw the vectors to visualise the angle better.



We can split up α into 90° + β and calculate β using a triangle.



$\therefore \tan\beta = \frac{1}{1} = 1 \quad \therefore \beta = \tan^{-1}(1) = 45° \quad \therefore \alpha = 90 + 45 = 135°$

c) Equation for velocity with constant acceleration : $\underline{v} = \underline{v}_0 + \underline{a}t$

where v_0 (u) is the initial velocity when t = 0.

$\therefore v(t) = (3i - 4j) + (i - j)t \quad \therefore v(T) = (3 + T)i + (-4 - T)j$

The velocity vector at time T must be a scalar multiple of $(11i - 13j)$ so the ratio of their i components must be equal to the ratio of the j components.

$\therefore \frac{3 + T}{11} = \frac{-4 - T}{-13} \quad \therefore -39 - 13T = -44 - 11T \quad \therefore 2T = 5$

$13T - 11T = 44 - 39 \qquad T = 2.5$ seconds //

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

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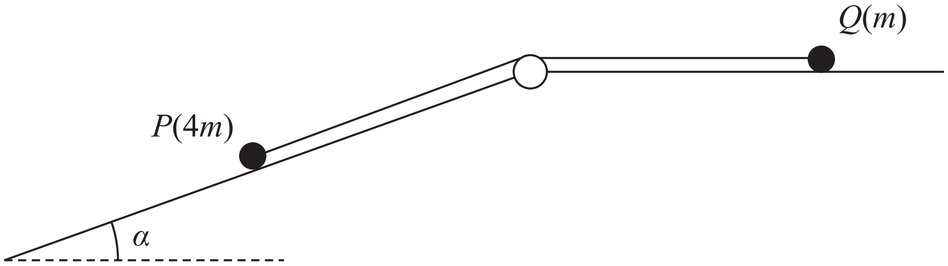


Figure 4

A particle P of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The particle P is attached to one end of a light inextensible string.

The string passes over a small smooth pulley that is fixed at the top of the plane. The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 4, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$

For the motion before Q reaches the pulley

(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

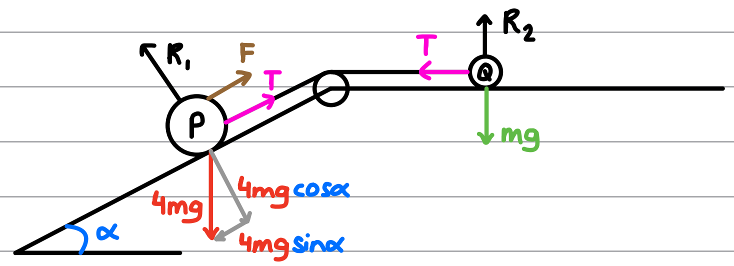
(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

a) Draw the diagram labelling forces. $\tan\alpha = \frac{3}{4} \therefore \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$



Using $\Sigma F = ma$ for the forces on Q which contribute to its acceleration:

$T = ma$ //

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Question 7 continued

b) We can use the formula $\Sigma F = ma$ to set up equations to find the acceleration which can be used to find the tension.

$\Sigma F = ma$ for particle P with down the slope as positive :

$\therefore 4mg\sin\alpha - F - T = 4ma$ Using $F = \mu R$: $F = \frac{1}{4}R_1 = \frac{1}{4} \times 4mg\cos\alpha$
$= mg\cos\alpha$

$\therefore 4mg\sin\alpha - mg\cos\alpha - T = 4ma$

$\therefore T = 4mg\sin\alpha - mg\cos\alpha - 4ma$ where $R_1 = 4mg\cos\alpha$ since the forces perpendicular to the slope must be equal in magnitude as there isn't any movement in this direction.

We can now equate this with the tension equation from part a) .

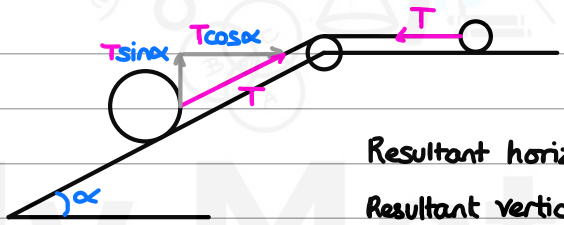
$\therefore 4mg\sin\alpha - mg\cos\alpha - 4ma = ma$

$\therefore 5ma = 4mg\sin\alpha - mg\cos\alpha$

$5ma = 4mg\left(\frac{3}{5}\right) - mg\left(\frac{4}{5}\right)$

$5a = \frac{12}{5}g - \frac{4}{5}g = \frac{8}{5}g \quad \therefore a = \frac{8}{25}g \quad \therefore T = ma = m \times \frac{8g}{25} = \frac{8mg}{25}$ //

c) Redraw the diagram but only include forces applied on the pulley.



Resultant horizontal force (positive left) : $T - T\cos\alpha$

Resultant vertical force (positive up) : $T\sin\alpha$

$\therefore$ Magnitude of resultant force : $\sqrt{(T - T\cos\alpha)^2 + (T\sin\alpha)^2}$

$= \sqrt{\left(T - \frac{4}{5}T\right)^2 + \left(\frac{3}{5}T\right)^2} = \sqrt{\left(\frac{T}{5}\right)^2 + \left(\frac{3}{5}T\right)^2}$

$= \sqrt{\frac{T^2}{25} + \frac{9T^2}{25}} = \sqrt{\frac{10T^2}{25}}$

$= T \times \frac{\sqrt{10}}{5} = \frac{8mg}{25} \times \frac{\sqrt{10}}{5} = \frac{8mg\sqrt{10}}{125}$ //

d) Tension will be the same throughout any section of the string.

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Question 7 continued

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Question 7 continued

Q7

(Total 13 marks)

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8. [*In this question* **i** *and* **j** *are horizontal unit vectors directed due east and due north respectively and position vectors are given relative to a fixed origin.*]

A ship A moves with constant velocity $(3\mathbf{i} - 10\mathbf{j})$ km h^{-1}

At time t hours, the position vector of A is $\mathbf{r}$ km.

At time $t = 0$, A is at the point with position vector $(13\mathbf{i} + 5\mathbf{j})$ km.

(a) Find $\mathbf{r}$ in terms of t.

(2)

Another ship B moves with constant velocity $(15\mathbf{i} + 14\mathbf{j})$ km h^{-1}

At time $t = 0$, B is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ km.

(b) Show that, at time t hours,

$$\overrightarrow{AB} = [(12t - 10)\mathbf{i} + (24t - 10)\mathbf{j}] \text{ km}$$

(4)

Given that the two ships do not change course,

(c) find the shortest distance between the two ships,

(6)

(d) find the bearing of ship B from ship A when the ships are closest.

(2)

a) Formula for position : $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ $\therefore$ $a = (13i+5j)+(3i-10j)t$

b) Using the above formula, we can also make an equation for Ship B.

$b = (3i-5j)+(15i+14j)t$ Using the formula : $\overrightarrow{XY} = y - x$

$\overrightarrow{AB} = b - a$ $\therefore$ $\overrightarrow{AB} = (3+15t)i + (-5+14t)j - [(13+3t)i + (5-10t)j]$

$= (3-13+15t-3t)i + (-5-5+14t+10t)j$

$= (12t-10)i + (24t-10j)$ km

c) We can modulus $\overrightarrow{AB}$ to find the time at the minimum distance.

$\therefore |\overrightarrow{AB}| = \sqrt{(12t-10)^2+(24t-10)^2}$

$\therefore \overrightarrow{AB}^2 = (12t-10)^2+(24t-10)^2 = 144t^2-240t+100+576t^2-480t+100 = 720t^2-720t+200$

The minimum distance occurs when the derivative of the quadratic is zero.

$\therefore \frac{d}{dt}(720t^2-720t+200) = 1440t-720 = 0$ $\therefore$ $t = \frac{720}{1440} = 0.5$

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Question 8 continued

We can now substitute the value of t to find the minimum distance.

$\therefore |AB| = \sqrt{(12(0.5)-10)^2 + (24(0.5)-10)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$ km //

d) The bearing is the angle of AB relative to north, measured clockwise.

@ $t = 0.5$, $AB = -4i + 2j$

Now we find the angle θ that AB makes with the positive j-axis (north).

$\tan\theta = \left|\frac{\text{east component}}{\text{north component}}\right| = \left|\frac{-4}{2}\right| = 2 \quad \therefore \theta = \tan^{-1}(2) \approx 63.43°$

Since the east component is negative and the north component is positive, AB points north-west. However bearings are measured clockwise from north:

$\therefore$ Bearing $= 360° - \theta = 360° - 63.43° = 296.57° \quad \therefore$ Bearing $= 297°$

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Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END